

BOUNDARY CONDITIONS FOR HEAT TRANSFER THROUGH GLASS FURNACE BRICKWORK

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A method for calculating the boundary conditions for heat transfer through glassmaking furnace brickwork is presented. The average heat-transfer coefficients and heat fluxes for the principal structural elements of a furnace are determined. A database for mathematical modeling the thermal operation of furnaces is formulated.

Key words: glassmaking furnace, boundary conditions, heat transfer, heat flux.

The coefficient of heat transfer K , $W/(m^2 \cdot K)$, which establishes a unique relation between the resulting heat flux density q , W/m^2 , and the interior temperature t_{int} , $^{\circ}C$, is a given in external heat transfer calculations performed by the resolvent zonal method for the surface zones of the brickwork in the working space of a furnace [1].

Mixed boundary conditions are used to determine the heat-transfer coefficient. Boundary conditions of the first kind are set by the temperature t_{int} on the interior surface of the brickwork. Boundary conditions of the third kind are set on the exterior surface of the brickwork by the total heat-emission coefficient α_{ext} , $W/(m^2 \cdot K)$, and by the temperature of the ambient medium $t_{\text{amb}} = 40^{\circ}C$.

Starting from the formulation of the boundary conditions the equation of heat transfer through multilayer brickwork has the form

$$q = K(t_{\text{int}} - t_{\text{amb}}), \quad (1)$$

where

$$K = \frac{1}{\sum_{i=1}^n R_i + \frac{1}{\alpha_{\text{ext}}}}; \quad (2)$$

R_i is the thermal resistance of the i th layer of the brickwork, $(m^2 \cdot K)/W$, and n is the number layers in the brickwork.

In turn

$$R_i = \frac{S_i}{\lambda_i(t)}, \quad (3)$$

where S_i is the thickness of the i th layer of the brickwork, m , and $\lambda_i(t)$ is the temperature dependence of the thermal conductivity of the i th layer of the brickwork, $W/(m \cdot K)$.

For natural convection on the exterior surface of the brickwork the coefficient α_{ext} of total heat emission is calculated from the equation

$$\alpha_{\text{ext}} = k_{\text{surf}}(t_{\text{ext}} - t_{\text{amb}})^{0.25} + \frac{\sigma_0 \varepsilon_{\text{surf}}}{t_{\text{ext}} - t_{\text{amb}}} \left[\left(\frac{t_{\text{ext}} + 273}{100} \right)^4 - \left(\frac{t_{\text{amb}} + 273}{100} \right)^4 \right], \quad (4)$$

where the coefficient k_{surf} takes account of the position of the surface and direction of heat emission in space: we take $k_{\text{surf}} = 2.4$ for the side walls of the working space and the melting tank of the glass furnace (vertical surfaces) and $k_{\text{surf}} = 3.3$ and 1.6 , respectively, for the horizontal surfaces with heat emission directed upward and downward (the furnace roof and tank bottom); $\varepsilon_{\text{surf}}$ is the emissivity of the external surface of the brickwork: $\varepsilon_{\text{surf}} = 0.8$ for brick brickwork and $\varepsilon_{\text{surf}} = 0.8$ and 0.2 , respectively, for steel sheet and surfaces covered with bright aluminum paint; t_{ext} is the exterior temperature of the cladding, $^{\circ}C$; and, $\sigma_0 = 5.67 \times 10^{-8} W/(m^2 \cdot K^4)$ is the Stefan–Boltzmann constant.

The gird of the side walls of the melting tank is forced air cooled. For $t_{\text{ext}} = 100 - 400^{\circ}C$ the overall heat-emission coefficient is given by the expression

$$\alpha_{\text{ext}} = (9.5 + 0.07t_{\text{ext}})(1 + 2w), \quad (5)$$

where w is the velocity of the cooling air, m/sec . We take $w = 50 m/sec$ [2].

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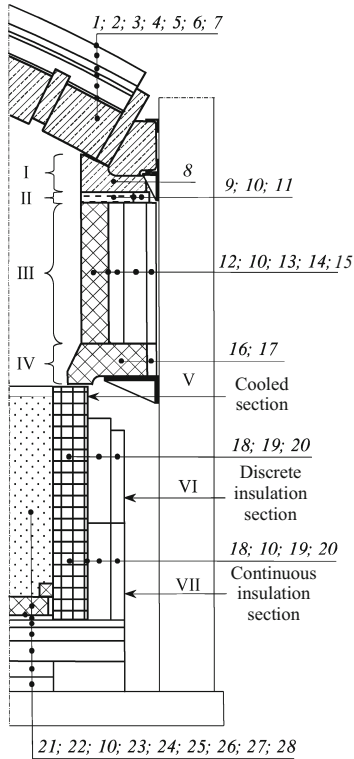


Fig. 1. Fragment of the transverse section of a glassmaking furnace: 1) DSU brick — 0.4 m; 2) STELLAMUR GLS mastic (0 – 0.3 mm grain) — 0.01; 3) LEGRAL 55/05 brick — 0.114 m; 4) STELLIT GH mixture (0 – 1 mm grain) — 0.030 m; 5) MKRV-200 mullite-silica felted fabric — 0.060 m; 6) KPD-400 foamed diatomite brick — 0.114 m; 7) STELLAMUR GLS mastic — 0.005 m; 8) DSU — 0.5 m; 9) Z-64 — 0.4 m; 10) RESIMUR ZM 322 mastic (0 – 0.5 mm grain) — 0.005 m; 11) MD-1450 — 0.114 m; 12) AZS-33 block with normal contraction cavity — 0.2 m; 13) MD-1650 brick — 0.114 m; 14) MD-1450 brick — 0.181 m; 15) KPD-400 foamed diatomite plate — 0.067 m; 16) AZS-33, NC — 0.5 m; 17) MD-1450 — 0.067 m; 18) AZS-37 block with reduced contraction cavity — 0.25 m; 19) MD-1450 — 0.183 m; 20) KPD-500 foamed diatomite brick — 0.116 m; 21) molten glass — 1.17/1.60 m (melting/fining zone, respectively); 22) AZS-33 plate with no contraction cavity — 0.12 m; 23) RESISTIT ZM 260 mixture (0 – 3 mm grain) — 0.03 m; 24) Z-64 — 0.065 m; 25) BN-60 plate — 0.1 m; 26) BN-40 block — 0.2 m; 27) MD-1450 — 0.13 m; 28) KPD-500 — 0.130/0.065 m (melting/fining zone, respectively).

Taking account of the temperature dependence of the thermal conductivity of the brickwork materials and the overall heat-emission coefficient, the problem of determining the heat flux into the surrounding medium is solved by the method of successive approximations (method of iterations). The iterative process is stopped when

$$\left| \frac{q_{\text{ext}} - q_{\text{thr}}}{q} \right| < \varepsilon_{\text{err}},$$

where q is the heat flux density determined from the relation (1); $q_{\text{ext}} = \alpha_{\text{ext}}(t_{\text{ext}} - t_{\text{amb}})$ is the density of the heat flux

density from the exterior surface of the brickwork into the surrounding medium; q_{thr} is the density of the heat flux passing through a wall with thermal conductivity

$$q_{\text{thr}} = \frac{t_{\text{int}} - t_{\text{ext}}}{\sum_{i=1}^n \frac{S_i}{\lambda_i(t)}},$$

where ε_{err} is the prescribed computational error, $\varepsilon_{\text{err}} = 1.0 \times 10^{-3}$.

An accurate calculation of heat transfer through a multi-layer wall presupposes a validated temperature given on the interior surface of the brickwork. As a rule, the heating temperature of surfaces is taken from experimental data, which in the first place are too sparse and in the second place can have a very arbitrary relation to the furnace being designed. In this connection it is desirable to use the results of numerical modeling of external heat transfer with optimal plume length [3].

The melting part of the furnace is divided into separate structural elements. The roof and longitudinal and end walls are singled out in the working space. The melting tank is calculated in greater detail. This pertains to the side walls, which are divided along the height into three computational sections, and the bottom of the tank [4]. The tank is divided along its length into a melting zone (70% of the tank length) and a fining zone. The forced air cooled surfaces are also singled out. The structure of the brickwork (see Fig. 1) and the characteristics of the refractory and heat-insulating materials are prescribed (Table 1) for each surface.

The nonuniformity of the temperature field on the surfaces of the working space and the identical structure of the brickwork in the structural elements which have been singled out give a basis for using in the calculation of thermal conductivity the average interior temperature \bar{t}_{int} . In application to the structure of a melting tank with a horseshoe flame the values of \bar{t}_{int} can be determined from two-dimensional equations approximating the temperature distribution on the surfaces of the working space of the furnace [3]:

– crown in the working space

$$t_{\text{cr}}(x_1, y_1) = 1399.65 + 511.682x_1 - 279.713y_1 - 327.564x_1^2 + 52.049x_1y_1 + 82.726y_1^2; \quad (6)$$

– right-hand longitudinal wall (heated side of the furnace)

$$t_{\text{long, rh}}(x_1, z_1) = 1313.145 + 761.654x_1 + 490.176z_1 - 542.979x_1^2 - 275.787x_1z_1 - 1646.567z_1^2; \quad (7)$$

– left-hand longitudinal wall (outlet side of the furnace)

$$t_{\text{long, lh}}(x_1, z_1) = 1278.097 + 422.825x_1 + 408.297z_1 - 208.091x_1^2 - 128.85x_1z_1 - 1567.367z_1^2; \quad (8)$$

TABLE 1. Thermal Conductivity and Working Temperature of Refractory and Heat-Insulating Articles

Article brand	Working temperature, °C	Thermal conductivity, W/(m · K)
DSU, GOST 3910–75	1600	$0.93 + 0.707 \times 10^{-3}t$
STELLAVUR GLS (95.5% SiO ₂)	1680	–
STELLIT GH (85.5% SiO ₂)	1400	$0.29 - 0.1 \times 10^{-3}t + 0.3 \times 10^{-6}t^2$
LEGRAL 55/05 (90% SiO ₂ , $\rho = 0.59$ g/cm ³)	1500	$0.15 + 0.2 \times 10^{-3}t - 0.04 \times 10^{-6}t^2$
MKRV–200, GOST 23619–79	1150	$0.04 + 0.1 \times 10^{-3}t + 0.08 \times 10^{-6}t^2$
KPD–400, TU 5764–002–25310144–99	950	$0.095 - 0.757 \times 10^{-3}t + 0.25 \times 10^{-6}t^2$
Z–64 (64% ZrO ₂)	1630	$2.7 - 0.01 \times 10^{-3}t + 0.077 \times 10^{-6}t^2$
MD–1450 (52% Al ₂ O ₃ , $\rho = 0.8$ g/cm ³)	1430	$0.23 + 0.1 \times 10^{-3}t$
MD–1650 (73% Al ₂ O ₃ , $\rho = 1.0$ g/cm ³)	1430	$0.384 + 0.07 \times 10^{-3}t$
AZS–33 (33% ZrO ₂)	1700	$8.84 - 11.9 \times 10^{-3}t + 7.0 \times 10^{-6}t^2$
AZS–37 (37% ZrO ₂)	1700	$6.0 - 5.63 \times 10^{-3}t + 3.86 \times 10^{-6}t^2$
BN–40 (43% Al ₂ O ₃)	1350	$1.02 + 0.62 \times 10^{-3}t - 0.3 \times 10^{-6}t^2$
BN–60 (62% Al ₂ O ₃)	1550	$1.379 + 0.32 \times 10^{-3}t$
RESISTIT ZM 260 (18% ZrO ₂)	1650	$2.5 - 1.8 \times 10^{-3}t + 0.8 \times 10^{-6}t^2$
RESIMUR ZM 362 (31% ZrO ₂)	1650	$5.0 - 5.1 \times 10^{-3}t + 2.32 \times 10^{-6}t^2$

– end wall (flow-through)

$$t_{e,ft}(y_1, z_1) = 1575.210 - 14.301y_1 - 350.668z_1 + 24.909y_1^2 - 361.626y_1z_1 - 398.718z_1^2, \quad (9)$$

where x_1, y_1, z_1 are, respectively, $x/L_f, y/L_f$ and z/L_f ; L_f is the length of the working space of the furnace, m.

Averaging Eqs. (6) – (8) gives: $\bar{t}_{cr} = 1477.9^\circ\text{C}$, $\bar{t}_{\text{long, rh}} = 1526.7^\circ\text{C}$, $\bar{t}_{\text{long, lh}} = 1434.0^\circ\text{C}$ and $\bar{t}_{e,ft} = 1526.7^\circ\text{C}$, respectively. For the inlet end wall we set $\bar{t}_{e,in} = 1355.2^\circ\text{C}$ [3].

Working space of the furnace. We shall now examine the computational results for the parameters of heat transfer through the brickwork of the working space of a furnace.

Crown (Table 2). Modern heat insulation is installed on the main part of the surface of the crown. The low value of the heat-transfer coefficient 0.49 W/(m² · K), due to the high thermal resistance of the brickwork 1.99 (m² · K)/W, makes it possible to decrease the heat losses into the ambient medium from 4786.4 to 705.4 W/m², i.e., 6.8-fold.

The expansion joints between crown sections and the interlocking row of the brickwork are left uninsulated because of safety considerations. The area of the uninsulated surface is 7.44% of the crown area. Even though the heat losses through the uninsulated surface of the crown are high, the average coefficient of heat transfer \bar{K} is characterized by a low value 0.70 W/(m² · K), while the average heat flux \bar{q} does not exceed 1009.0 W/m².

Longitudinal walls of the working space (Table 3). The computational results show that the average heat losses through the longitudinal walls depend strongly on the construction of the walls (see Fig. 1). About 40% of the area of a longitudinal wall falls on sections with uninsulated (I) and inadequately insulated refractory brickwork (II, IV). As a re-

sult, if the heat fluxes through the heat-insulated section of the heated and outlet sides of the furnace are 968.7 and 892.5 W/m², respectively, then the average value \bar{q} for the indicated sides reaches 2104.0 and 1916.3 W/m².

A detailed accounting of the structural features of a longitudinal wall makes it possible to determine a realistic value of the average coefficient of heat transfer. This value is > 2 times greater than the value of K for an insulated section of the wall.

End walls of the working space (Table 4). An end wall consists of a thermally insulated section (III) and a bearing baddeleyite-corundum tooth (IV). The structure of the thermal insulation of both sections is similar to that of a longitudinal wall (see Fig. 1).

The areas of the heat-insulated surface of the flow-through and inlet walls are 88.6 and 77.7% , respectively. In consequence, the average heat-transfer coefficient and heat flux are much lower than for a longitudinal wall. Neverthe-

TABLE 2. Computational Results for Heat Transfer through the Furnace Crown

Computational parameters	Insulated section of the crown	Uninsulated section of the crown
$\bar{t}_{\text{int}} = \bar{t}_{cr}, ^\circ\text{C}$	1477.9	1477.9
$t_{\text{ext}}, ^\circ\text{C}$	83.6	229.9
$R, (\text{m}^2 \cdot \text{K})/\text{W}$	1.99	0.26
$\alpha_{\text{ext}}, \text{W}/(\text{m}^2 \cdot \text{K})$	16.17	25.23
$K, \text{W}/(\text{m}^2 \cdot \text{K})$	0.49	3.33
$q, \text{W}/\text{m}^2$	705.4	4786.4
$\bar{K}, \text{W}/(\text{m}^2 \cdot \text{K})$	0.70	
$\bar{q}, \text{W}/\text{m}^2$	1009.0	

TABLE 3. Computational Results for Heat Transfer through a Longitudinal Wall of a Furnace (Heated/Outlet Sides)

Computational parameters	Section of the wall surface			
	I	II	III	IV
$\bar{t}_{\text{int}} = \bar{t}_{\text{long.rh}}/\bar{t}_{\text{int}} = \bar{t}_{\text{long.lh}}, ^\circ\text{C}$	1526.7/1434.0	1526.7/1434.0	1526.7/1434.0	1526.7/1434.0
$t_{\text{ext}}, ^\circ\text{C}$	226.1/215.7	169.2/162.8	106.9/102.7	224.2/211.6
$R, (\text{m}^2 \cdot \text{K})/\text{W}$	0.32/0.33	0.58/0.58	1.46/1.49	0.33/0.34
$\alpha_{\text{ext}}, \text{W}/(\text{m}^2 \cdot \text{K})$	21.64/20.98	18.14/17.76	14.48/14.22	21.52/20.72
$K, \text{W}/(\text{m}^2 \cdot \text{K})$	2.71/2.64	1.58/1.56	0.65/0.64	2.67/2.55
$q, \text{W}/\text{m}^2$	4030.2/3686.6	2344.2/2181.5	968.7/892.5	3966.2/3552.0
$\bar{K}, \text{W}/(\text{m}^2 \cdot \text{K})$		1.42/1.37		
$\bar{q}, \text{W}/\text{m}^2$		2104.0/1916.3		

less, it should be noted that \bar{q} and \bar{K} differ considerably from the values characterizing the heat-insulated sections of the walls.

Melting tank of the furnace. We shall now examine the computational results obtained for the parameters of heat transfer through the brickwork of a melting tank filled with dark-green molten glass. The temperature dependence of the effective thermal conductivity λ_{eff} of the molten glass is represented by the equation

$$\lambda_{\text{eff}} = 1.0908 - 0.5 \times 10^{-3}t + 6 \times 10^{-6}t^2. \quad (10)$$

Since heat is transferred into the molten glass through its exterior surface, the main boundary condition for the calculation is the distribution of the temperature t_{mg} on this surface. In two-dimensional dimensionless coordinates we have [3]

$$t_{\text{mg}}(x_1, y_1) = 1132.904 + 931.278x_1 + 67.537y_1 - 675.024x_1^2 + 4.718x_1y_1 - 130.010y_1^2, \quad (11)$$

where x_1, y_1 are x/L_{mt} and y/L_{mt} , respectively, and L_{mt} is the length of the melting tank, m.

TABLE 4. Computational Results for Heat Transfer through the End Walls of a Furnace

Computational parameters	Section of wall surface			
	Inlet wall		Flow-through wall	
	III	IV	III	IV
$\bar{t}_{\text{int}} = \bar{t}_{\text{e.ft}}/\bar{t}_{\text{int}} = \bar{t}_{\text{e.in}}, ^\circ\text{C}$	1355.2	1355.2	1525.9	1525.9
$t_{\text{ext}}, ^\circ\text{C}$	99.2	201.2	106.8	224.1
$R, (\text{m}^2 \cdot \text{K})/\text{W}$	1.51	0.35	1.46	0.33
$\alpha_{\text{ext}}, \text{W}/(\text{m}^2 \cdot \text{K})$	14.01	20.07	14.47	21.51
$K, \text{W}/(\text{m}^2 \cdot \text{K})$	0.63	2.46	0.65	2.67
$q, \text{W}/\text{m}^2$	829.7	3233.6	968.0	3962.4
$\bar{K}, \text{W}/(\text{m}^2 \cdot \text{K})$		1.04		0.88
$\bar{q}, \text{W}/\text{m}^2$		1364.8		1309.1

Let us transform Eq. (11) into the one-dimensional form

$$\bar{t}_{\text{mg}}(x_1) = 1236.3 - 30.112x_1 + 1274.6x_1^2 - 613.08x_1^3 - 539.06x_1^4, \quad (12)$$

where \bar{t}_{mg} is the average temperature of the molten glass surface over the width of the tank, $^\circ\text{C}$.

We obtain from Eq. (12) the average values of the surface temperature of the molten glass for the melting and fining zones: $\bar{t}_{\text{mg}} = 1369.1$ and 1415.3°C , respectively; we shall take these values as the boundary conditions for calculating heat transfer through the melting tank. The molten glass is treated as the first layer of a multilayer wall.

Tank bottom (Table 5). The computational results show that the proposed structure of the brickwork at the bottom of the tank gives it a high thermal resistance $R = 1.88$ and $1.43 (\text{m}^2 \cdot \text{K})/\text{W}$ and, in consequence, a low value of the heat-transfer coefficient $K = 0.51$ and $0.66 \text{ W}/(\text{m}^2 \cdot \text{K})$ for the melting and fining zones, respectively. The effectiveness of the thermal insulation of the bottom brickwork gives low heat losses into the ambient medium ($\bar{q} = 679.5$ and $914.1 \text{ W}/\text{m}^2$), as a result of which the temperature of the melt layer at the bottom is high $t_{\text{bot}} = 1297.4$ and 1286.6°C , corresponding to the conditions for high-capacity glass-making.

TABLE 5. Computational Results for Heat Transfer through the Melting Tank of the Furnace

Computational parameters	Melting zone	Fining zone
$\bar{t}_{\text{int}} = \bar{t}_{\text{mg}}, ^\circ\text{C}$	1369.1	1415.3
$t_{\text{bot}}, ^\circ\text{C}$	1297.4	1286.6
$t_{\text{ext}}, ^\circ\text{C}$	98.0	113.0
$R, (\text{m}^2 \cdot \text{K})/\text{W}$	1.88	1.43
$\alpha_{\text{ext}}, \text{W}/(\text{m}^2 \cdot \text{K})$	11.72	12.51
$\bar{K}, \text{W}/(\text{m}^2 \cdot \text{K})$	0.51	0.66
$\bar{q}, \text{W}/\text{m}^2$	679.5	914.1

TABLE 6. Computational Results for Heat Transfer through the Side Walls of the Melting Tank of the Furnace

Computational parameters	Melting zone			Fining zone		
	Wall section			Wall section		
	V	VI	VII	V	VI	VII
$\bar{t}_{\text{int}}, ^\circ\text{C}$	1369.1	1347.5	1316.6	1415.3	1368.4	1311.9
$t_{\text{ext}}, ^\circ\text{C}$	134.0	99.6	98.0	136.6	100.6	97.8
$R, (\text{m}^2 \cdot \text{K})/\text{W}$	0.06	1.50	1.51	0.06	1.49	1.51
$\alpha_{\text{ext}}, \text{W}/(\text{m}^2 \cdot \text{K})$	207.68	14.03	13.91	209.66	14.09	13.92
$K, \text{W}/(\text{m}^2 \cdot \text{K})$	14.68	0.64	0.63	14.72	0.64	0.63
$q, \text{W}/\text{m}^2$	19512.3	836.0	808.0	20235.1	854.0	804.1
$\bar{K}, \text{W}/(\text{m}^2 \cdot \text{K})$		2.91			2.56	
$\bar{q}, \text{W}/\text{m}^2$		3848.5			3478.8	

Side walls of the melting tank (Table 6). We shall take a linear temperature distribution over the thickness of the layer of glass. Then it is possible to determine the average heating temperature of the surfaces of the side walls of the tank at sections with discrete and continuous insulation [4]. For the cooled sections of the walls in the melting and fining zones we take $\bar{t}_{\text{int}} = \bar{t}_{\text{mg}} = 1369.1$ and 1415.3°C , respectively.

It follows from the data in Table 6 that even though the heat insulation of the walls is effective the heat losses through the cooled section have a dominating influence on the average heat-transfer coefficient and heat flux into the ambient medium. Since cooling intensity is dictated by the technological aspects of the work of the refractories [2], the heat losses through the gird of the tank can be seen as justified. Hence the values $\bar{K} = 2.91$ and $2.56 \text{ W}/(\text{m}^2 \cdot \text{K})$ (melting and fining zones, respectively) reflect the real conditions of heat transfer through the side walls of the melting tank.

In closing, we note that the values presented in this article for the average heat-transfer coefficients can be regarded as representative initial data for mathematical modeling of

the thermal work of high-capacity glassmaking furnaces. The computational results for the heat losses through the brickwork of the working space and melting tank can be used for constructing the heat balance of glassmaking furnaces with the energy-conserving structure of modern furnace enclosures.

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